Should You Take a Risk When You Do Not Know for Sure?

From Judging to Acting since Condorcet

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Summary

Condorcet proposed a principle of reasonable probability: actions entailing a prohibitive risk with a non-negligible probability should not be taken. This principle guides the development of knowledge as much as it guides the action itself. The mathematics developed by Laplace has allowed for the effective application of this principle in mathematical statistics (point estimates combined with a high confidence level) or in the management of insurance companies (calculating the loading rate to ensure the solvency of the company). During the same period, Tetens was developing related ideas – though with less mathematical efficacy. These ideas from the 18th century still apply today, both in (the interpretation of) certain modern decision models and in the informational and legal requirements that should be enforced to ensure that financial decisions are rational.

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The men of the Enlightenment sought to remove the arbitrariness from decision making by founding it on simple, robust principles. Jean Antoine Nicolas Caritat de Condorcet, and Pierre-Simon Laplace, for example, thought up a general criterion for validating judgements and founding action. During the same period, the philosopher Johannes Nikolaus Tetens showed how this criterion should lead to certainty, and thence to action. These ideas had such a profound impact on mathematical statistics that they became implicit, until reflection on the foundations of statistics brought to light the foundations laid by Laplace. The enduring relevance of the principle expounded by Condorcet was thus striking.

Condorcet's principle of "reasonable probability" makes it possible to construct arguments for decision making that resist uncertainty. These arguments are in keeping both with the etymology of the word "risk" and with modern portfolio theory. I start by describing the relation between the application of the "principle of reasonable probability" and standard normative decision theory. I then focus on the present-day application of Condorcet's ideas. The relevance of Condorcet today concerns not so much what one should do as what one can reasonably require.

1. The Enlightenment of Risk

During the modern era, the calculation of probabilities was initially developed as a rule of justice, until Blaise Pascal's wager and the *Port-Royal Logic* interpreted it as a decision rule. Controversy about the arbitrary nature of decision making led Condorcet to seek the indisputable foundations of a reasonable decision. The suitability of the expected value had been challenged by the *St. Petersburg Paradox*,[1] and the other rules proposed (Bernoulli's hypothesis, for example) appeared to be arbitrary in their exposition or specification (see for example Jallais–Pradier–Teira [2008]). At the end of the century, the search for an incontestable principle finally bore fruit and the immediate applications were numerous.

A. The principle of certainty

The concept of quasi-certainty, already documented in the 1660s,[2] was expressed by George Leclerc Buffon, who defined "moral impossibility" in the context of human mortality. Observing that a healthy, middle-aged man had a 1-in-10,000 chance of meeting with sudden death within the next 24 hours, and that no reasonable man gave the matter any serious thought, Buffon suggested that any probability lower than this should be regarded as zero.[3] Among other things, this provided an easy solution to the St. Petersburg paradox. Condorcet, on the other hand, preferred to speak of "strong confidence":[4] quite apart from the animosity between himself and the biologist,[5] Condorcet thought Buffon's position appeared to be too simple. Condorcet considered that several different categories of risks were relevant to economic decision:

> *A reasonable man should enter into business only if he finds a fairly high probability to get his investment back, with common interest and the price of his labour.*
>
> *Such a man will doubtless require a probability close to certainty of not losing his funds wholly (saving at least what is needed for the subsistence of his family); and also a very large probability of not diminishing his funds by more than a given amount.* (Condorcet [1784](1994), p. 486)

Thus, the *rational decision to enter into business* requires not only the moral certainty[6] of not going bankrupt, but also the improbability of there existing one or more prohibitive risks.[7] Together, these two criteria constitute the "principle of reasonable probability (to enter into business)".

Comparing the risk specific to two business affairs by comparing these respective probabilities should therefore provide a basis for decision. But this raises several problems. First, Condorcet – the inventor of the paradox that bears his name[8] – proposed three measures without indicating which of them was to be preferred in situations of indecision when each measure recommends a different strategy. Second, the determination of thresholds (particularly in the case of point estimation) does not appear to be based on any objective criterion. Condorcet proposed a series of conventional definitions founded on the observation of good practices (Jallais–Pradier–Teira [2008]). The development of the theory of thresholds (covering the ideas of both disaster threshold and confidence level) justifies Condorcet's position.

Today, levels of activity corresponding to bankruptcy and break-even points (disaster thresholds) are as well-accepted as confidence levels of 99% and 95%.

The problem of competition between thresholds is perhaps more recent. For Condorcet, the decision process starts with the exclusion of irrelevant strategies, those in which the risks are not of a reasonable *improbability*. We must then apply a criterion of decision making, such as the maximization of expectation, to the remaining strategies, those that satisfy the thresholds of "reasonable probability". This last step remains implicit in Condorcet, who concentrated more on removing the dominated prospects.

B. Condorcet: the promotion of insurance

The text in which Condorcet exposed his theory of thresholds in most detail was an article on marine insurance in the *Encyclopédie méthodique* that prompted further developments by Laplace and Sylvestre-François Lacroix.[9] In the mid-1780s, Condorcet was engaged in the intense promotion of insurance: he organized a prize awarded by the Academy of Sciences ([1783][10] 1994) and wrote several articles ([1784], [1785a], [1785b] 1994) and unpublished notes ([n.d.]).

Marine insurance, which had already existed for three centuries, had no need of such promotion, but agricultural insurance (which the author advocated) was only at the project stage. At the same time, in Latin countries, law and moral code both condemned the institution of life insurance that had already developed in England. If his apocryphal letters ([1785a], [1785b] 1994) popularized an adventurous new idea, the article "*Assurances maritimes*" in the *Encyclopédie méthodique* revealed Condorcet's faith in the very principle of insurance. Despite its misleading title, the article aimed to promote *all* forms of insurance. The argument Condorcet chose to convince his readers was based on economic calculation, applied both to the insurer and the potential policy-holders. Condorcet, like Daniel Bernoulli before him ([1731] 1982), brought both the insurance company and its customers into the equation; something that subsequent authors failed to do.

From an abstract point of view, the calculation is straightforward: Condorcet modelled a decision maker facing n identical operations, each of which can result in failure or success. The probabilities of the different levels of outcome for the insurer are therefore given by a binomial distribution. In practice, if the probability of failure is p, then the probability of obtaining m failures out of n draws is $C_n^m p^m(1-p)^{n-m}$. This does not give us an explicit distribution function, however. It is possible to cobble one together by writing: if X is the binomial variable designating the number of failures out of n draws, then $P(X \leq m) = \sum_{k=0}^{m} C_n^k p^k(1-p)^{n-k}$.

The calculation is all the more laborious since the unknown here is m: we want to calculate m for a given probability $P(X \leq m)$, corresponding to the desired confidence level.[11] We have to add together the terms of the binomial, which is very tedious, making Condorcet's article hard to read and severely limiting its practical application. The primary objective of the formalization is to determine the selling price of insurance so as to reduce the probability of the insurer going bankrupt to a morally negligible level. Condorcet brought a new perspective to decision making and its application in the *(mathematical) theory of risk*, with the aim of determining the loading rate necessary to protect the insurance company against the consequences of an accumulation of accidents. For lack of a suitable mathematical tool, Condorcet's success was only partial: he founded the principle, but the analytical complexity of the theory made it practically unusable.

When applied to business management in general, Condorcet's *principle of reasonable probability* can be summed up as follows: the rate of profit is set in such a way as to make solvency almost sure (or *morally certain*). Before presenting subsequent developments in the theory of insurance, it is worth noting that the author linked the question of risk to that of estimation. The second part of the article "*Assurances maritimes*" deals with the "probability of future events according to past events" (or, in the words of Laplace, the "probability of constant causes through [the study of] events already observed"). The author thereby returned to his works of the 1770s on the asymptotic Bayesian method, which he had developed with Laplace,[12] independently from Thomas Bayes, whom the French only learnt about later. Although the results of this Bayesian method were rather meagre,[13] it testifies to Condorcet's concern to take into account the *statistical* nature of data, to the contrary of that "frivolous object"[14] ("games of chance" characterized by *a priori* probabilities) to which "surveyors restricted themselves for so long" (before him, of course). Condorcet did not yet control the degree of confidence of the estimations; the analogous approach to situations of economic and statistical decision making was still only latent.

C. Tetens: the "fund risk" and its applications

At about the same time as Condorcet, Tetens was working on the same problems: the mathematical theory of risk and the question of estimation. Tetens constructed a measure of risk, the *Risico* (sometimes called the *Risico der Casse* to indicate that it referred to the risk incurred by an insurance fund). This measure was neither the average error,[15] as Karl Borch thought ([1969], p. 1), nor the linear average risk,[16] as Bohlmann–Poterin du Motel wrote ([1911] 1993, p. 577). The subtleties of Tetens' writing were too much even for the patience of his admirers. Inspired by the German combinatorial school, he used polynomials to describe the random variables, and his probability calculus was therefore based on perplexing algebra theorems. To start with (§18–19), Tetens discussed the expected value of deviations for outcomes below the mean.[17] He illustrated the subject with the example of a die numbered from 0 to 5. The expected value of a variable defined in this way is $\frac{5}{2}$, the outcomes below the mean are 0, 1 and 2, and the deviations are therefore $\frac{5}{2}$, $\frac{3}{2}$ and $\frac{1}{2}$. As the outcomes are equiprobable (out of six possibilities), the risk indicator for this lottery is:

$$\left(\frac{5}{2}+\frac{3}{2}+\frac{1}{2}\right)\times\frac{1}{6}=\frac{3}{4}.$$

In the case of symmetrical lotteries – in other words, where the deviations from the mean on either side of the expected value are equal, as they are when throwing dice – the average error (divided by two) and the risk indicator are obviously equal, but not the linear average risk, because there are no negative results. If we subtract from the random variable its expected value – the entry price to the game – then the three measures are identical. In one particular case, therefore, we can formally demonstrate that Tetens' indicator of dispersion is equal to other indicators that were introduced later. We must not forget, however, that *in his demonstrations*, Tetens did not limit himself to zero-sum lotteries, that is, lotteries that satisfy the condition of equality with other indices. The commentators' error may stem from the fact that his *examples* are always of this type, since life annuities are sold on their expected value.

If Tetens has remained in the memory of the actuarial profession for having developed a concept (and a measure) of fund risk, I must nevertheless stress the particular nature of the use to which he put it. Whereas Condorcet, Laplace and Lacroix had no difficulty in accepting the necessity for *loading*, both to cover the insurer's costs and to safeguard his solvency (Condorcet [1784] 1994; Laplace [1812] 1993, pp. 439-440; Lacroix [1821], p. 233), Tetens rejected such a practice:

> *We see [thanks to the risk indicator] what the guarantee to produce represents. He who assumes the guarantee cannot, by the nature of the thing, demand anything for this, any more than a player starting a game of chance with another can demand something without gaining an advantage. He may equally well lose as win, and he must simply ask himself whether he is prepared to gamble such a large sum.*[18]

So for Tetens, there could be no exception to the principle of justice governing risky decision making, reaffirmed countless times since Pascal (see Jallais–Pradier [1997]). This point of view resonated with the public nature of German institutions: with such principles, insurance companies could not be profitable! Nor could they lay any claim to solvency! They therefore had to be subsidized and guaranteed by the state. In this context, it is easy to understand why these calculations were of interest both to the public authorities (which provided both the market and the subsidy) and potential contractors. This detail deserves attention, with all due respect to Max Weber, for here we find French Catholic partisans of state control more at ease with affairs of money than their German Lutheran cousins.

If Tetens' *Risico* did not serve to calculate the loading, then of what use was it? Tetens' idea may have come from a suggestion by Abraham de Moivre, who wrote, in his *Doctrine of Chances*:

> *6. The Risk of losing any sum is the reverse of Expectation; and the true measure of it is the product of the Sum adventured multiplied by the Probability of the loss.* (Moivre [1756] p.4)

Tetens was clearly influenced by Moivre. His use of Stirling's formula was accompanied by an explicit reference to Moivre (contrary to 18th century usage) [1730].[19] Tetens' developments on the subject could simply be treated as mathematical exercise with the aim of extending this concept of risk to more complex random variables than the Bernoulli variables considered by Moivre. And indeed, the abstraction of Tetens' work (notably in a detour by way of generating functions), shows a real interest in pure questions of algebra and suggest that it could be an exercise in style. Nevertheless, the philosopher's simultaneous interest in "fund risk" and the estimation risk leads us to look for another interpretation.

D. The estimation risk

Tetens sought to establish the value of mortality tables used to calculate life annuities. He started with the premise that a minimum of 1000 observations should be considered to obtain an "error" (on the expected value) of less than half a year.[20] He therefore studied by analogy the risk of estimation with the risk of action. This analogy raises two problems. First, variability is not studied in a satisfactory manner: there is a confidence interval, but it is not associated with a probability. If we calculate retrospectively the probability implicit in Tetens' method, it appears to be far too weak to be ascribed any "moral certainty".[21] Second, the interpretation of Tetens' result is uncertain: given the use that is made of mortality tables, should there be 1000 observations *for each age group* (and gender), for the tables to be accurate at each age? The author provided no solution to these problems.

These results therefore appear symbolic: the author points out the problem of statistical induction without really tackling it. The reader might object that by presenting it without any of the accompanying calculations, we may have distorted Tetens' argument. But the complexity of these computations necessitates a full presentation elsewhere.[22] These developments may be rather exotic, but they prove that Condorcet was not the only thinker to perceive an analogous connection between knowledge and action, subject to the same principle of reasonable probability. On this matter, the *Théorie analytique* went a step further by providing the possibility of controlling the level of probability of the risk in the action.

E. Actuarial methods: a mathematical theory of risk

At a time when the English pragmatists were not troubling themselves about dispersion (insurance company results) or the sampling errors of the mortality tables (Pradier [2003]), Laplace, on the contrary, was tackling Condorcet's problem in order to model the stakes. The chapter of *Théorie analytique* dedicated to the question of "profits dependent on the probability of future events" starts by recalling the framework of his predecessor's reflections.[23] He presents identical operations leading to binary results (success/failure), and therefore a binomial draw. "Laplace's method" consists in a normal approximation for binomial variables.

The probability $P(X \leq m) = \sum_{k=0}^{m} C_n^k p^k (1-p)^{n-k}$ is therefore approximated by

the integral $\frac{1}{\sqrt{\pi}} \int_{-\infty}^{\frac{m-np}{\sqrt{2npq}}} exp\ (-v^2) dv.$

This does not appear to constitute a simplification, but insofar as only the upper integration limit changes, it makes it possible to use a table of the Laplace–Moivre law (such as that produced by Kramp [1799]) to obtain the values of the integral without calculation. Condorcet's model thus became usable. It was possible to calculate rapidly the loading necessary for a given safety level. And that was not all. The nature of the tool also allowed Laplace to make refinements: "Laplace's method" ([1785] 1878-1912) was based first on an analytic approximation, not on a theorem of convergence in probability (Laplace's famous "central limit theorem" [1810] 1878-1912). Instead of only considering identical binary draws (success or failure), the author started by introducing the possibility of different variables (binomials of which the probabilities or consequences can change [1812] 1993, p. 430), and then multinomial draws ([1812] 1993, p. 432). Finally, and this point might appear to Condorcet's readers as a homage to the departed elder, Laplace determined the laws of future events according to past events ([1812] 1993, p. 434). In other words, he too proposed a Bayesian estimation of frequencies, instead of the *a priori* probabilities with which the insurance literature habitually contented itself.

At this point, one might be tempted to think that Laplace's contribution to the question studied by Condorcet lies solely in the use of this normal approximation. But the Norman extended Condorcet's idea even further. By permitting more complex random variables to be taken into account,[24] he could envisage the *effective* determination of the loading rate that he was seeking to justify.[25] The same concern for effectiveness guided his research into statistical estimation. A series of works on demography (Laplace [1781], [1786] 1878–1912) had made it possible to define the "method", the analytic approximation which, as we have seen, played such an instrumental and crucial role in the integration of safety conditions. Let us see exactly how Laplace used the mathematics developed for statistical estimation – for what we might call tests of hypotheses – in his insurance calculations. In the tests of hypotheses,[26] the reasoning is based on the observation of a *binary* qualitative characteristic (gender in Laplace [1781] 1878–1912, the fact of being born within the year[27] for Laplace [1786] 1878–1912); the distributions considered are therefore binomial, once again. These tests of hypotheses consist in seeking to determine the probability of a real frequency being far-removed from its estimation on a sample. The analytical form of the problem is the study of a variable for which the probability law is of the form[28]

$$\frac{C_{p'}^{q'}\int_0^1 x^q(1-x)^{p-q}x^{q'+1}(1-x)^{p'-q'}dx}{\int_0^1 x^q(1-x)^{p-q}dx}.$$

Approximating this quantity then makes it possible to use the same approximation to study any binomial variable. It would be wrong to think that the general problem of dispersion (which applies in questions of insurance) is explicit in Laplace. What leads the author to use the same tools is the *mathematical* analogy between identical functional forms, and not a conceptual type of analogy, which would consist in seeking domains of application for a pre-existing theory of dispersion.[29]

Let us therefore conclude by observing that Laplace applied Condorcet's *principle of reasonable probability* as a criterion of management for economic activities and as a criterion of moral certainty in statistical estimation. The particular interest of this latter point is that it led to the subsequent crystallization of the conventional probability thresholds of mathematical statistics yet to be developed.

2. The Legacy of Laplace

Laplace's work is dazzling both by the generality of the link he established between knowledge and action under uncertainty and by the precision of his solution to the questions raised by Condorcet. After Laplace, developments in the mathematical theory of risk were centred on the dimension of management rather than its epistemic foundations. The work of Sylvestre-François Lacroix (Pradier [2003]) provides a clear example of this focus on practical aspects. The contributions of Lacroix, Laplace and Tetens to decision theory fell into obscurity: Tetens was only known to actuaries,[30] and it seemed as if Laplace was only of interest to geodesists, actuaries[31] or historians of science. In fact, the principles of Condorcet have shaped mathematical statistics, and we only have to scrape a little to discover the Laplacian bedrock under the fertile soil.

A. The speculations of finance

In an article published in 1952, Andrew Roy stressed the fact that financiers wanted rough and ready rules of thumb rather than abstract theories. To meet this need, he considered a programme of income maximization with a safety constraint: he required a very high probability (95%) that the income should exceed a given minimum.[32] And so we come back to Condorcet's *principle of reasonable probability*, here baptised the *safety-first principle*. There are two differences, however, from Condorcet's model. First, the conventional safety level was fixed with reference to statistical tests of general use. Practice has therefore created habits. Second, instead of considering the return distributions directly, Roy simplified the calculations by using the Bienaymé–Chebyshev inequality.[33] Ultimately, the economic decision was therefore expressed in terms of mean and variance, since Roy plotted the curve representing the optimum pairs in terms of return and risk, producing the analytical expression of what is called, after Harry Markowitz [1956], the "efficient frontier". Unfortunately, as Peter Bernstein observed [1992], Roy had the bad luck to see his article come out […] three months after the publication of Markowitz's article in the

Journal of Finance. His contribution, so directly Condorcetian, was therefore eclipsed by the work of the US financier.

Markowitz is sometimes considered the father of the "mean-variance" theory of risky decision making, based on a representation of random variables in terms of moments. By uncovering the origin of these representations, Pradier [2000] showed the extent to which the Laplacian legacy was concealed in Markowitz's works. And yet comparison with Roy brings home their common point of departure: the principle of reasonable probability to undertake something. Markowitz sought to resolve the following problem: given a number of financial securities (shares), each characterized by a return (measured by the mean) and a level of risk (the variance), how can we find the portfolio of maximum return for a given level of risk, or the portfolio of minimum risk for a given level of return (these two programmes are actually linked; they are said to be *dual* in programming)? The mathematical difficulty comes from the "quadratic" nature of the programmes to be solved: variance is the expected value of the square of the deviation from the (expected) mean, and it is much more complicated to manipulate squares of variables than their *linear* functions (that is, without powers). So the real "innovation" of Markowitz does not lie in a representation of risk preferences, but in the solution of a problem of pure calculation: how to optimize a quadratic function under linear constraints. At the beginning of the 1950s when Markowitz was writing, there could be no hope of effectively applying the analysis to real data: it took the power of all the army's computers to calculate the optimum portfolios with just 50 different securities!

Even if it still appeared inapplicable, Markowitz's analysis gave a mathematical interpretation of the very old principle that "you should not put all your eggs in one basket". The solution of his programme involved the choice of diversified portfolios (rather than investing everything in one sole security). There was nothing particularly new in this: Nicolas Bernoulli ([1731] 1975) had already summed up his cousin Daniel's article in those very words! On the other hand, the reason leading to diversification in Markowitz's work is worth examining: it is the imperfect correlations between securities that explain why it is usually better to diversify. For example, when petrol prices rise, oil companies are favoured, while shares in car manufacturers are penalized. Typically, these securities are correlated negatively, and it may therefore be worthwhile holding both of them at the same time. Even assets that are correlated positively, but not completely, will offset each other's fluctuations to some extent, though not wholly. The Condorcetian foundation is thus conjured away, but it is waiting to resurface in the agricultural applications of Markowitz's model.[34]

B. *O fortunatos agricolas*

If people usually dwell on the debt that agricultural economists owe to Markowitz, the pioneering work of Rudolf Freund [1956] also deserves attention. As the title of his article – "The introduction of risk into a programming model" – indicates, he tackles the same technical problem as Markowitz [1956] (and Simon [1956]). Freund chose to maximize a combination of mean and variance (therefore a non-linear function) under constraints that were themselves linear. Once again, Markowitz was neither unique nor, visibly, the first. In this case, he could at least exploit the general theory of choice expressed in terms of moments of random variables, whereas Freund used a particular utility function. There is reason to believe that Markowitz's advantage derived above all from his position at the RAND Corporation, and from that of his thesis supervisor at the Cowles Foundation – both institutions were held in the highest esteem. In comparison, the name of North Carolina State College (where Freund wrote his thesis) is more likely to inspire condescendence: it was a college, not a research institution. There was thus an undeniable effect of institutions in the reputation granted to Markowitz.

Furthermore, agricultural economics did not enjoy great visibility: Freund proposed an application of the Markowitz model to the determination of optimal crop programming for a farm as early as 1956 – two years before what is considered the "founding" article by Tobin. And yet Freund is only ever cited by agricultural economists. Freund [1956] did not construct the efficient frontier, as Markowitz did, but derived the optimal "crop portfolio" directly from a utility function for the first two moments of total income. In the absence of efficient frontier, Freund's model is less elegant than that of Markowitz, but this weakness also ensured *compatibility with the expected utility theory*, which Markowitz – subjected to criticism from theoretical economists – took 30 years to achieve. In any event, specialists in agricultural economics, largely impervious to changing fashions, made great use of the Markowitz model, and raised a good number of questions about its theoretical status.

Discussion about Freund's model shows how little interest in theoretical unification there was among agricultural economists. They did not seek to construct a theoretical frame of reference like the one Jan Mossin, William Sharpe and John Lintner gave to finance. In fact, application of Markowitz-style formulations raised two problems: the efficient frontier did not offer a direct reading in terms of safety, and its calculation was too complex, as we have seen.

Jean-Marc Boussard [1969] promoted the first argument by showing that the important thing was not optimizing but knowing how to optimize. For a farmer trying to determine his crop plan, risk takes the form of an income below a given threshold. This is exactly the formulation of Condorcet–Roy, whom the authors do not cite, however. As in the case of Tetens and Condorcet in the 18th century, who worked in parallel without knowing it, it is worth investigating the reasons for this simultaneity. The answer clearly lies in the integration of Laplacian mathematical statistics into the culture of engineers (including agricultural engineers). On the theoretical level, the agricultural models certainly lose something of the generality of the Markowitz analysis, and the efficient frontier disappears, detracting from the "elegance" of the result. In its place, a limit threshold is specified (analogous to the Value at Risk (VaR) in finance), which is perfectly suitable for the purpose. One can then observe farm production plans under the hypothesis that the farmers behave in accordance with the model and with the specified parameters (price, VaR).

Wanting to describe decision-making mechanisms in farming is not the only motivation that has led economists to take liberties with the Markowitz model. During the same period, Peter Hazell remarked that the Markowitz model was too demanding with regard to the quality of information and its processing. On the first point, the author observed that it is difficult to calculate a variance–covariance matrix, whereas one can propose some estimations of returns.[35] On the second point, he observed that solving a quadratic programme requires the use of a powerful computer (for reasons of precision[36]), whereas a linear programme needs no more than an operator trained in the simplex method.[37] Hazell therefore proposed replacing the minimization of variance in the Markowitz programme by the minimization of the absolute mean deviation (of income), whence the acronym MOTAD (*Minimization Of Target Absolute Deviation*). As an alternative to semivariance, Hazell proposed to consider only the *negative* mean deviation, and proved that the calculations remained just as simple.[38]

The choice of simultaneous simplicity and robustness brings us back to the fundamental considerations of Condorcet. I can illustrate these heuristics that are robust against uncertainty by means of an original reflexive example.

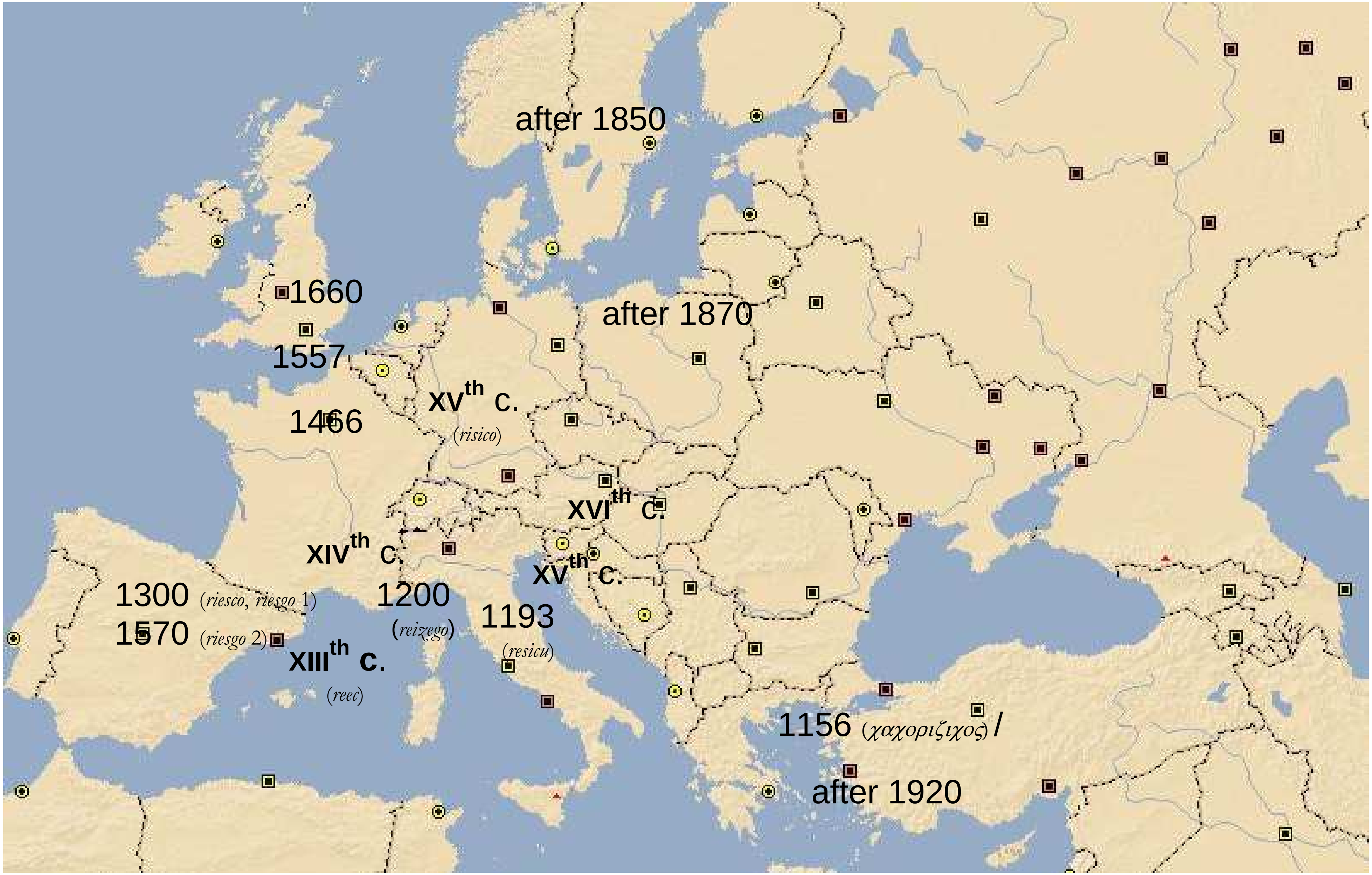
C. The genesis of risk (a little diversion)

Up until recently, the very origin of the concept of risk was uncertain. Etymological dictionaries proposed a dozen different hypotheses (Pradier [1998], chap. 1), and yet the authoritative authors (Niklas Luhmann and Max Weber, to name just two of the eminent German social scientists) presented one sole thesis. Thus, in 1990, the great German sociologist Niklas Luhmann wrote that the concept of risk appeared at the beginning of the modern age "to indicate a problematic situation that cannot be described with sufficient precision by the existing vocabulary" (Luhmann [1990], p. 10). By way of new problematic situation, we think of nothing less than the great discoveries, religious reforms, the emergence of capitalism, and the development of modern science, thereby linking these phenomenal forms with Western development, as Max Weber ([1908] 1985) did. But two recent texts suggest that we should reconsider this view of the medieval heritage.

Sylvain Piron [2004] unearthed the right historical material, allowing him to identify the Arabic syllable *rizq* in the cargoes of Pisan merchants leaving the port of Bugia: through them *resicum* was transmitted throughout the whole of Christendom. So risk followed the same route as Arabic numerals, during the same period and in a roughly identical cultural environment (only the merchants, for whom counting was an everyday activity, found an interest in these Arabic numerals that simplified calculations). In his article, Piron recounts the improbable story of his discovery: although the Arabic origin of the word had been argued for by Marcel Devic as early as 1863, a succession of morphological and semantic objections had blocked the evidence. It was a series of articles published between 1999 and 2002 that convinced Piron, while the philologists had been erring for more than a century. In the end, it was serendipidity that led the tireless researcher to the right source.

In 1998, when textual material was just beginning to be digitized and before publication of the pertinent articles cited by Piron, I had proposed a completely different heuristic, but which turned out to be quite robust to the sampling risk. Instead of relying on the origin of the word, I thought it would be more reliable to start with what we actually knew (the first documented appearances of it). By consulting all the etymological dictionaries in the reading room of the Richelieu National Library, not forgetting the invaluable Boiteux [1968], I was able to produce the map shown below. This map clearly shows how *risk* appeared in European languages from the starting point of Italy, and that it spread along trade routes. As this is also the case for epidemics, fashions and religious reforms, one might argue that this spread tells us nothing about the original connotation of the term "risk". It did tell us, however, that the early spread of the word followed a maritime path: it moved at lightning speed along the Tyrrhenian coast, slightly less fast in the Adriatic, but no less than along the other great trade routes. Risk was therefore a word of the merchant sailors, as Piron proved:

> *This progression makes the transition from the Arabic* rizq *to the Latin* resicum*, as a close synonym of* fortuna*, perfectly credible. The essential nuance introduced by the neologism, and which no doubt explains its success among Italian notaries, lies in the fact that it allows imputation to a legal subject. We can observe this* a posteriori *in the respective specialisation of the two terms in commercial documents. As a general rule,* fortuna *refers to divine providence, whom one hopes will grant a favourable outcome to the voyage, whereas* resicum *applies to the backer assuming the financial consequences of the operation. The two concepts are nevertheless very closely linked, and there is nothing implausible in the idea that Italian merchants adopted an Arabic term equivalent to* fortuna *but better adapted to the needs of commercial documents* [Piron, 2004, p. 10].


after 1850
1660
after 1870
1557
XV^th c.
1466
(risico)
XVI^th c.
XIV^th c.
XV^th c.
1300 (riesco, riesgo 1)
1200
1570 (riesgo 2)
(reizego)
1193
XIII^th c.
(resicu)
(reec)
1156 (χαχοριζιχος) /
after 1920

This general formulation (the neologism as an expression of new uses) is reminiscent of Luhmann's, but here it is applied to those Middle Ages that brought us not only Arabic numerals and the premises of probability calculus (Meusnier [2004]), but also the basic financial techniques of double-entry bookkeeping, the bill of exchange and insurance. All that remained was to acquire the appropriate tools to understand the "risk" phenomenon. And this is exactly what the modern age brought, with the development of scientific method.

This example of little consequence illustrates (though obviously without proving) the idea of heuristics that is robust against uncertainty. All Piron's predecessors failed, because they could not find the documents to prove their hypotheses. This confirms the intuition that the probability of finding them (by chance) was very low. The heuristic of searching among what is already known, on the contrary, did not make it possible to trace the term "risk" back to its original source, but to characterize its diffusion in a very precise way. This example makes it possible to introduce a first element of comparison between contemporary decision theories and the principle of reasonable probability (as mentioned in the section on farmers): the information requirement. Let us now define this concept more precisely and examine the comparison, so as to better understand the status of Condorcet's principle.

3. The Present-Day Relevance of Condorcet

With the development of decision theory over the last 60 years, the principle of reasonable probability has been subjected to pitiless, critical scrutiny. It has been discredited for *normative* reasons. It can, however, be proved that it corresponds to a possible interpretation of the rational decision, and that its application is not as unreasonable as the theorists would have had us believe. Finally, Condorcet's point of view reflects more than ever the minimum requirements of the rational decision.

A. Economic decision making: normative and descriptive theories

The examples borrowed from finance and agricultural economics demonstrated above show that the neo-Condorcetian theories[39] have a descriptive relevance in applied economics. They describe as much the decision-making process as they do the result of agents' decisions. And yet they appear to stand in contradiction to the theoretical models, which are based on the theory of expected utility: axiomatized since the mid-1940s, this is considered the *normative theory of reference*.[40] In what way is it normative? In that it excludes the irrational behaviour that other theories tolerate: in particular the Dutch book and the money pump. These expressions describe the possibility of exploiting inconsistent choices to extort from reputedly irrational agents the price of their irrationality, either statically (Dutch book: a decision maker is offered a set of bets, the subjective probabilities of which are not additive) or dynamically (money pump – a series of choices leading a decision maker who does not maximize his expected utility to display intransitivities). These sophisticated general categories find common expression in the *Borch counterexample* [1969]. With the help of an example, Karl Borch showed that it is impossible to have reasonable preferences within a mean-variance framework: Markowitz and Roy showed that it was a possible representation of the principle of reasonable probability.

If the theory of expected utility is normative to the point of showing that the other theories are unreasonable, the demonstration is not wholly convincing. The Allais paradox [1979] opened the way to the work of psychologists, such as Daniel Kahnemann, Paul Slovic and Amos Tversky. They have shown how prevalent violations of the "normative" theory are. In fact, as early as 1952, Maurice Allais developed an experimental system to disprove the normative character of the theory empirically (Jallais–Pradier [2005]). As a consequence, the constraints imposed on decisions by expected utility are only justified by a tautological definition of normativity. The aim of this argument was not to restore to prominence the theories inherited from the principle of probability on the grounds of their greater descriptive pertinence: advances made in the representation of behaviour led to the emergence of a new family of what are called *rank-dependent utility* theories, even further removed from the ideas of Condorcet. If the Condorcetian theories are unsuitable for representing agents' choices, they are nonetheless an effective tool for representing the decision-making processes themselves.

As we have seen with the agricultural economists, the dearth of available information often prohibits the use of an expected utility decision model due to lack of inputs. We can express this informational superiority of the principle of reasonable decision making, notably by recalling the effectiveness of variance as a statistic (Fisher [1920]), or by showing that the measures of risk inherited from the normative theory are more prone to error than variance is (Pradier [1998], chapter 6). Examples abound of decisions made from information that is more limited and more difficult to manipulate (than that presupposed by the normative theory). We can cite, in no particular order: the validity of marine insurance premiums calculated for centuries by rule of thumb, the statistical efficiency of standard deviation, and on the contrary, the difficulty of estimating functions, or simply of estimating under uncertainty the law of a random variable for which it would be more plausible to determine a minimum, and finally the Value-at-Risk used in finance. A cursory reading might then suggest that the neo-Condorcetian theories, although subject to fatal errors, are better adapted to day-to-day life and the uncertainty that governs it. This presentation nevertheless deserves to be examined in greater detail.

B. Why metaphysical unreason is physically reasonable

The first thing we should do is limit the opposition between expected utility and neo-Condorcetian theories. According to Allais [1979], there was a Homeric battle during the 1950s between the advocates of a descriptively accurate decision theory and the defenders of the more normative "American school". Allais invented the "American school" (said to include, among other "Americans", Edmond Malinvaud, Bruno de Finetti, Georges-Théodule Guilbaud and Georges Morlat) to imply the existence of an *École française*, of which he would, logically, be the leader, after Pascal, Laplace, Poincaré and certain others. This opposition is a fiction (Jallais–Pradier [2005]). It is more interesting to look for the areas of compatibility between the theories, and then reduce the remaining divergences. One path consists in considering that the decision maker's utility function leads to moments being treated as parameters of decision: this is notably the case for quadratic utility functions, the properties of which were explored in the 1950s.[41] Unfortunately, the hypothesis of increasing risk aversion on which they are founded is not plausible;[42] so much so that Markowitz himself, who had initially considered this path, finally abandoned it[43] (except for Taylor series approximations). Another solution consisted in only considering normal variables,[44] until Meyer proposed a more general criterion of equivalence based on the comparison of *densities*: the "location and scale condition".[45]

On the strength of these premises, one might conclude that the choice between normative theory and neo-Condorcetian theory is a matter of context or interpretation. The normative theory guarantees abstract properties: normative qualities, analytical determination of an (optimal) solution. The neo-Condorcetian is intuitive and corresponds to the idea of decision making in a context of limited rationality: once one has ensured one or more levels of safety, the selection of a strategy is no longer necessarily a maximization, but a choice between dominant alternatives (dominating those that do not provide the required level of safety). The problem with this representation of decision making is that the choice of a strategy is based on criteria that are not clearly defined, which excludes the modelling of agents' behaviour. One can then specify the principle of reasonable probability in a *Markowitzian* way, admittedly not completely satisfactory (because of Borch's counterexample) but nevertheless admissible, because the empirical relevance of the counterexample is doubtful. In real decision-making situations, the variables of decision (the returns) generally possess a "regular" shape (Day [1965], Levy–Markowitz [1979], Kroll–Levy–Markowitz [1984], Pope–Ziemer [1984]). The Condorcetian theory could therefore be opposed by contrived counterexamples, but it is effective in action. Expected utility theory, on the other hand, although it represents a perfectly rational decision rule, is not always followed by the decision makers, either because it cannot account for certain behaviour (the Allais paradox, among others), or because it requires a quality or a processing of information of which the agents are incapable. Setting the metaphysical possibility (of the counterexample) against the physical impossibility, as means to justify a simplistic and robust theory, had already been proposed by Jean le Rond D'Alembert (D'Alembert [1761], Rieucau [1998]).

4. The Problems of the 21st Century: Macro-robustness to Risk

Is it not fruitless to seek to rehabilitate a *principle of reasonable probability* that is expressed, in the final analysis, in the VaR? After all, the VaR has contributed to the current crisis and shown how easily it can be manipulated (Galichon [2008]). Moreover, the development of limited liability has changed the point of view of economic decision makers about their own affairs. Insofar as they are no longer exposed to the threat of ruin, they are inclined to take more risks. If we add to that the increase in the size of companies allowed by the joint stock company, and the disconnection between ownership and control, the problems of economic regulation appear to be insoluble: as we have seen since the summer of 2007, the big risks play strategically on the public authorities, exerting threats illustrated by the recurrence of the expression "too big to fail". Clearly, the Condorcetian analysis appears powerless in the face of these macroeconomic problems, all the more so since the marquis had no conception of the idea of moral hazard, as evidenced by his totally unsustainable project of *agricultural insurance*. Does that mean that the Condorcetian model has lost all relevance?

The answer is no, because it would be illusory to think that the reduction of aggregate risk can be achieved without the vigilance of individual decision makers. Individual vigilance to risk is therefore a prerequisite, and the duty of the government is first to establish a cognitive and institutional framework that allows for the attainment of individual rationality. While there is no institution in Condorcet's model, present-day economic activity is exercised through complex mediations: contributors of capital, whose liability is limited, do not make the day-to-day management decisions, and accounting rules do not necessarily enforce disclosure of the information required for decision. Reference to Condorcet's decision model is nevertheless important, because it makes it possible to understand the information necessary for rational decision: in particular, historical values and market values must coincide, and probabilistic hypotheses must be solid.

Obviously, these two conditions are not satisfied when market conditions are extreme, as we have seen recently. Ideally, the regulator should be able to enforce a return to historical values and "suspend" market-value accounting when the liquidity of instruments becomes insufficient (it should be possible to make this decision for a given category of instrument). Likewise, verifying the validity of probabilistic hypotheses is not far-removed from the idea of *ad hoc stress tests* decided by the market authorities, although it should be easier to trigger such episodes. The idea that LTCM's losses were due to an exceptional deviation – as exceptional as the contagion of subprime defaults – is clearly based on an erroneous probabilistic representation. Generally speaking, the purely probabilistic modelling of default risk is a source of trouble that Irving Fisher had clearly identified as long ago as 1906: "The high risk not only makes the terms of the loan onerous, but these onerous terms make the uncertainty of repayment greater, and so on in a vicious circle" (Fisher [1906], p. 406). Consequently, the scores must be truly discriminating, and there is good reason to suspect the banks of strategic "too-big-to-fail" threats.

Thus, the regulations should provide the decision maker with the conditions of possibility of the Condorcetian decision, but this is not sufficient, because the moral hazard linked to the socialization of losses must also be restricted. Here, the problem is not so much to prevent the abuse of credulous investors by wrongdoers, but to avoid the macroeconomic consequences of behaviour that is perfectly rational on an individual level. In this matter, we are all the more discomfited since the benevolent planner has been a fantasy from the Age of Enlightenment through to Ramsey: nobody knows what each person should do to contribute to the general harmony. All that remains is to manage the strategic behaviour of agents. We can see just how far this management is imbued with discretion in the absence of regulation. Condorcet's model remains an ideal that should be made possible.

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Notes

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[1] A more complete presentation is given in Jallais–Pradier [1997]. Let me just specify that the *St. Petersburg Problem* consists in determining the amount one should bet to enter the following game: "A throws a coin into the air. B undertakes to give him 1 pound if tails comes up on the first throw, 2 pounds if it only comes up on the second throw, 4 pounds if it does not come up until the third throw, 8 pounds for the fourth throw, and so on". The expected value for player A (denoted by C) is:

$$C = \frac{1}{2}\times 1 + \frac{1}{2}\left(\frac{1}{2}\times 2 + \frac{1}{2}\left(\frac{1}{2}\times 4 + \frac{1}{2}\left(\frac{1}{2}\times 8 + \frac{1}{2}(\ldots)\right)\right)\right) = \frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 4 + \frac{1}{16}\times 8 + \ldots$$

in other words

$$C = \sum_{i=0}^{+\infty} 2^{-(i+1)} \times 2^{i} = \sum_{i=0}^{+\infty} 2^{-1} = \sum_{i=0}^{+\infty} \frac{1}{2} = +\infty .$$

[2] According to Rieucau [1997] this expression was relatively widespread when Buffon made his own specific use of it. In his *Introduction à la philosophie* (1736, p. 128), let us note that 's Gravesande [1736] considers it to be in "common" use. In this respect, O.B. Sheynin [1977] reports that it can already be found in the writings of authors such as Descartes, Huyghens and Leibniz, as well as Jean and Nicolas Bernoulli. We could add the *Port-Royal Logic* to the list. Here, however, I shall use the term in the sense defined by Buffon and respected by Condorcet.

[3] Buffon [1777], p. 38: "As no man of this age (fifty-six), when reason has acquired all its maturity and experience all its strength, has any fear of suddenly dying in the next twenty-four hours, although the odds that he will not die during this short lapse of time are only ten thousand (...) to one; I conclude that any probability less than or equal to this should be regarded as zero."

[4] Condorcet [1785c], p. 23.

[5] On the hatred between the two figures, see Rieucau [1995], p. 21 and especially note 85.

[6] See Rieucau [1998]. Condorcet ([1784] 1994, p. 492) proposed "[…] to learn how, in practice, men who are considered wise and whose projects succeed, have solved the same problem; for example, what has been the probability of not losing that which the insurers have obtained in the different insurance offices which have been able to carry on their trade and prosper." Condorcet was thus proposing a truly *experimental definition* of rationality.

[7] By envisaging the probability of the events: "obtaining a normal profit from his activity", "not losing more than a certain sum" and "not losing all his possessions", Condorcet distinguished three "events" of which the first includes the second, and the second includes the third. The risk inherent in economic activity therefore admits of three levels: "working for nothing", "losing a considerable sum in business dealings" and "going bankrupt" (these are the complementary events to those described by Condorcet).

[8] We know that the word *paradox* was not Condorcet's (nor even Jean-Charles de Borda's), although the marquis did concede that the content was "paradoxical". On this subject, see Rieucau [1997], Ch II, section 2, § 3, n. 102.

[9] See the works of Pierre Crépel [1988] and Jean-Nicolas Rieucau [1998].

[10] This prize, proposed in 1781 by the Academy of Sciences at Condorcet's instigation, was not awarded on the date originally planned (1783) and was twice postponed. Lacroix and Charles François de Bicquilley finally shared the prize in 1787. It is therefore likely that at this date, at least, Laplace had not made progress on the subject.

[11] The reader wishing to consult the calculations in their entirety will find them in the original text, or in Crépel [1988].

[12] Bernard Bru and Pierre Crépel [1994] pp. 256–60 *n.* have greatly elucidated the question of the relations between Bayes, Condorcet and Laplace.

[13] Here, the principle of "Bayesian" estimation involves considering that if m ships have foundered and n have survived, the probability ("of future events according to past events") of success is $\frac{m+1}{m+n+2}$. See Condorcet [1784], p. 491; Laplace [1774].

[14] Condorcet [1785c], p. 601.

[15] Let there be a variable X of expected value $\overline{x}$, presenting n outcomes x_i, $i \in [1, n]$ ranked in increasing order of value, of which the respective probabilities are p_i, then there exists a larger n_0 such that $\forall i \leq n_0$, $x_i \leq \overline{x} = 0$. The absolute mean deviation is written $\sum_{i=1}^{n} p_i |\overline{x} - x_i|$.

[16] Using the same notation, the linear average risk is written $\sum_{i \leq n_0} p_i x_i$.

[17] The risk of Tetens is therefore $R = \sum_{i \leq n_0} p_i |\overline{x} - x_i|$.

[18] §38. Unless otherwise specified, the references concern *Dritte Abhandlung — Versuch über das Risico der Casse bey Versorgungsanstalten* by Tetens [1786].

[19] Moivre [1730] is developed further in Moivre [1756], "A method of approximating the Sum of Terms of the Binomial $(a+b)^n$ expanded into series, from whence are deduced some practical Rules to estimate the Degree of Assent which is to be given to experiments", pp. 243 *et seq.* Moivre's title is such that one may rightly wonder whether this mathematician was not behind the analogy observed in Laplace and Tetens. However tempting this hypothesis may be, it must be recognized that Moivre, though responsible for some indisputable analytical developments (we speak of the Moivre–Laplace law), maintained a very "Bernoullian" approach. It was not a question of the inversion of probabilities (and of the theorem of Jacques Bernoulli), or of dispersion, but on the contrary of convergence towards the mean, the expression of Providence: "*Altho' Chance produces Irregularities, still the Odds will be infinitely great, that in process of Time, those Irregularities will bear no proportion to the recurrency of that Order which naturally results from* ORIGINAL DESIGN."

[20] See Pradier [1998], pp. 89 *et seq.* and 119 *et seq.*

[21] If we keep the hypothesis (valid for the examples) that $R =$ average error, Tetens was therefore interested in the interval $[\mu - R, \mu + R]$. As the average error here is 0.8 times the standard deviation, we have:
$[\mu - R, \mu + R] = [\mu - 0{,}8 \cdot \sigma, \mu + 0{,}8 \cdot \sigma]$, where σ denotes the standard deviation. Obviously, we can calculate this probability with the help of a normal distribution table. We obtain 0.5762 (for the probability of a standard normal distribution lying within an interval of 0.8 times the standard deviation on either side of the mean). We are therefore far-removed from moral certainty.

[22] See Pradier [1998]. We can, however, note the two fundamental features: first, Tetens reduced the outcome of life annuities to a binary result (either the annuitant dies on the day he takes out the policy, or he receives his annuity right through to the end of the table) which leaves the expected value unchanged; second, he

approximates the risk of the sum of annuities, identified with a binomial, by using the Moivre version of Stirling's formula.

[23] Except that, as Crépel [1988] pointed out, the insured party had disappeared from scholarly concerns. Furthermore, Laplace did not cite Condorcet, but this neglect was customary in the 18th century.

[24] Taking into account multinomial variables makes it possible to account for *life annuities*, the outcome of which cannot be reduced to success or failure, as the potential lifespan of the annuitant generally covers a fairly broad range (one usually does not subscribe to a life annuity policy on the eve of old age). To simplify the calculations, Laplace introduced an "analytical adjustment" to the mortality tables. For example, he chose to take the ratio between the number of people still alive at age x (y_x) and the initial cohort (y_0) as being equal to $1-\frac{x}{n}$, where n is the age at which the last member of the cohort dies. This is equivalent to considering a linear decrease in the population, known in the actuarial literature as "Moivre's hypothesis" [1725] (see for example Bohlmann–Poterin [1911], p. 508). Thanks to these hypotheses, he was able to extend his calculations to cases of joint annuities on several lives ([1812], pp. 436–7).

[25] In particular, Laplace [1812] p. 439 justified the loading of annuities: "I will simply observe that all those establishments must, to prosper, *reserve themselves a profit* [our italics] and considerably multiply their number of deals, so that, their real profit becoming almost certain, they are exposed as little as possible to the great losses that could destroy them." The author concluded on p. 440: "Thus, in the case of an infinite number of deals, the real profit of the business becomes *certain and infinite* [our italics]. But then those who deal with this establishment suffer a mathematical disadvantage, which must be compensated for by a moral advantage, the appreciation of which will be the subject of the next chapter [on moral expectation]."

[26] The term is obviously anachronistic, but Laplace's method is perfectly applicable today. The two differences from the modern method of constructing tests of hypotheses are the following: first, Laplace used an analytical approximation and not a probabilistic convergence theorem (which allowed him to add variables following different laws); second, he approximated the sums of variables by means of the law that bears his name, whereas today we use a standard normal distribution (the Laplacian law has a standard deviation of $\frac{1}{2}$). Broadly speaking, the relevance of Laplace's work on estimation has justified the interest of contemporary statisticians such as William Cochran [1977], pp. 158-60, [1978], or of course Bernard Bru [1988].

[27] See Bru [1988], in particular notes 101 and 105, pp. 37–8.

[28] This is a beta function, meaning a function of the form $B(p,q)=\int_0^1 x^{q+1}(1-x)^{p+1}dx$. Leonhard Euler [1781] made a detailed study of it (published in1794), which is why Adrien-Marie Legendre called it the "Euler integral of the first kind".

[29] This opinion may seem contentious, inasmuch as it constitutes a counterexample to the thesis of Giorgio Israël [1996], but for the time being this counterexample remains an isolated case.

[30] See Bohlmann, or Godfrey Hardy in his lectures to the Institute of Actuaries in 1908.

[31] An important review of the literature was presented by Harald Cramér [1930].

[32] See Boussard [1969].

[33] The Bienaymé–Chebyshev theorem makes it possible to write that the probability of deviating from the mean is lower than a fraction of the variance. Thus, for a variable *X* with mean *m* and a distance *d*, the theorem gives: $P(|x-m| \geq d) \leq \frac{\sigma^2}{d^2}$. Minimizing the probability that the return will be lower than a given threshold is therefore equivalent to minimizing the variance of the portfolio for a given return, as Roy did [1952].

[34] There is a vast literature in this domain, the status of which is problematical. On this point, see Pradier [1998], pp. 231–4.

[35] Hazell [1971], pp. 51 wrote : "Under certain circumstances, the farmer may have subjective values for certain parameters and prefer to base his decisions on these subjective evaluations. But this type of information is difficult to obtain for a complex activity, and it seems unlikely that the variance–covariance matrix could be completely specified."

[36] Hazell [1971], p. 56 n.: "The [computer] programmes available suffer severely from rounding-off problems." Hazell also mentions the problem of singular matrices.

[37] Hazell [1971], p. 57: "This [minimising the mean deviation] can easily be done by following the linear programming model."

[38] *Ibid.*, pp. 59–60.

[39] This expression is a quick way to designate the models founded on Condorcet's principle of reasonable probability: Roy's *safety-first* model, Markowitz's *mean-variance* (and extensions to more numerous moments), Hazell's *MOTAD*, and so on.

[40] Expected utility theory represents risky decisions as the application of a utility function with random prospects, for which one therefore calculates the quantity $\sum_{i=1}^{n} p_i u(W + x_i)$ where *W* represents the agent's level of wealth, and the random variable *X* is described by the family (p_i, x_i) of probabilities associated with values. Although it is expressed by a decision function apparently identical to that of Bernoulli [1731], the expected utility theory is based on different foundations; in particular the function *u* can be distinguished from the certain utility function for wealth (Bouyssou–Vansnick [1990]).

[41] See the heated discussion between Jacob Marschak and Allais on the occasion of Paul Samuelson's speech at the colloquium of Paris (Samuelson *et al.* [1952], pp. 151–3. Markowitz [1959], p. 287 takes up this point of view.

[42] The *absolute aversion* to risk is the ratio $-\frac{u''(R)}{u'(R)}$, where *u* represents the utility function and *R* the agent's income. See Pratt [1964], p. 132. It makes it possible to evaluate the premium that an agent (maximizer of expected utility) is prepared to pay to get rid of an additive risk (that is, a random variable added to his income); in short, the sum that an individual is willing to pay to exchange the lottery $(R+X)$ for the prospect $(R+E(X))$, where *X* is a zero-expectation random variable, and *R* is the agent's initial wealth. This premium is very nearly equal to half the risk variance multiplied by the absolute aversion. It seems obvious that, the greater the agent's wealth, the lower the premium: the absolute aversion should therefore *decrease* with wealth.

[43] See for example Kroll–Levy–Markowitz [1984].

[44] This is the case in the literature from Markowitz [1958] to Baron [1977].

[45] Two variables represented by their probability distributions F and G respect the location and scale condition if there exists a pair of real numbers (α,β) (α positive) such that, for all x, $F(x)=G(\alpha x+\beta)$. See Meyer [1987].